Non-Perishable Stochastic Inventory Model with Reworks.

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Abstract- We considered two stores in the system one for fresh items and another for returned items. Most of the classical inventory models assume that all items manufactured are of perfect quality. However, in real-life production systems, due to various controllable and/or uncontrollable factors the generation of defective items during a production run seems to be inevitable and they should be reworked. In this paper, we considered that defective items will dysfunction before expire date, a service will be provided once it returns to the service center. If the store of rework items is full then the next case will be served at home as early as possible. The arrival of demand for fresh items and for rework items follows Poisson process with parameter λ and δ . From fresh items store, items will be provided to the arrival customer within a negligible service time. When inventory level for fresh items reaches to s an order takes place which follows exponential distribution with parameter γ. When inventory level is zero then arrival customer will be lost forever. The objective of this research is to develop a mathematical model to derive some system characteristics and to investigate the effect of cost function for the production systems. A suitable mathematical model is developed and the solution of the developed model using Markov process with Rate-matrix is derived. Also the systems characteristics are numerically illustrated. The validation of the result in this model was coded in Mathematica 5.0.

Index Terms- Inventory, Non-perishable, Stochastic Model, Re-order, Markov Process, Replenishment, Reworks

1 INTRODUCTION

Return policy is one of the most important challenge in the customer driven business world. By return policy we understand a contract between the manufacturer and forward positions in the supply chain (retailers, suppliers, customers), concerning the procedure of accepting back products after having sold them, either used or in an as-good-as-new state. Customer returns of as-good-as-new products have increased dramatically in the recent years. Growth in mail-order and transactions over the Internet has increased the volume of product returns as customers are unable to see and touch the items they decide to buy, so they are more likely to return them. Several studies draw attention to possible causes for high number of returns: in 2007, Americans returned between 11 and 20% of electronic items, which adds up to the staggering amount of $13.8 billion, out of which just 5% were actually broken. The rest failed to meet the customers' expectations. Most often the customers discovered that the product they had bought did not have the functionality they expected. The way management handles return items plays an important role in the company's strategy to success, especially in the area of e-commerce.

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2 LITERATURE REVIEW

This chapter fills the need for a comprehensive and up-to-date review of research on managing non-perishable inventory in the area of operations management, especially a review that can show the recent trends and point out important future research directions from the perspective of operations management and supply chain management. We concentrate on the research done mainly on stochastic inventory management and on those papers which, in our view, are important and lay the foundation for future work in one of the directions we detail. We also refer to some papers on non-perishable items in supply chain management literature to put the research in perspective.

In a single-stage production system, a certain number of defective items results due to various reason including poor production quality and material defects and subsequently a portion of them may be scrapped as well. Depending on the portion of defectives, if number of defective items raises then the optimal batch size varies depending on several cost factors such as setup cost, processing cost and inventory holding cost. So the production system may have a repair or rework facility at which the defective items will be rework and/or corrected to finished products. In a production

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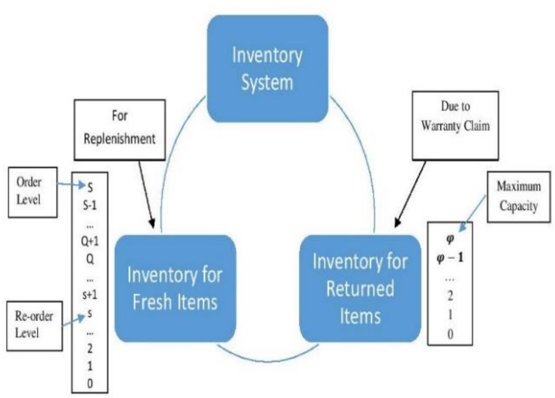
system where there is no repair or rework facility, defective items go to scrap. These defective items are wasted as scraps at each stage in every production cycle and as a result many industries having no recycling or reworking facility lose a big share of profit margin.

Recent developments in this field may be found in the work of **Huel-Hsin Chang et al. (2010)** where they studied the optimal inventory replenishment policy as well as on the long-run production inventory costs. A little attention was paid to the area of imperfect quality EPQ model with backlogging, rework and machine breakdown taking place in stock piling time. **Chung (2011)** developed a supply chain management model and presents a solution procedure to find the optimal production quantity with rework process. Chiu,Y.S.P. et al. developed a Mathematical modeling for determining the replenishment policy for a EMQ model with rework and multiple shipments. **Brojeswar Pal et al. (2012)** developed a multi-echelon supply chain model for multiple-markets with different selling seasons and the manufacturer produces a random proportion of defective items which are reworked after regular production and are sold in a lot to another market just after completion of rework. **Krisnamoorthi et al. (2013)** developed a single stage production process where defective items produced are rework and two models of rework processes are considered, an EPQ without shortages and with shortages **C.K.Sivashankari, S.Panayappan(2014)**proposed a Production inventory model where they consider reworking of imperfect production, scrap and shortages.

3 Mathematical Model

3.1 Figure Of The Model



3.2 ASSUMPTIONS

a) Initially the inventory level for fresh items is S and for return items is φ .

b) Arrival rate of demands follows poisson process with parameter λ for fresh items and δ for return items.

c) Lead-time is exponentially distributed with parameter γ for fresh items.

d) If the inventory of fresh items is in φ then service for the return items will be promptly at customer's home.

e) Service will be provided for the return items with exponential parameter μ.

3.3 NOTATIONS

a) S→Maximum inventory level for fresh items.

b) φ→Maximum inventory level for returned items.

c) λ → Arrival rate of demands for fresh items.

d) δ →Arrival rate of demands for returned items.

e) γ →Replenishment rate for fresh items.

f) μ →Service rate for returned items.

g) I(t) →Inventory level at time t for fresh items.

h) $E = E_1 \times E_2$ →The state space of the process.

i) x(t) → Inventory level at time t for retuned items.

j) E_1={0, 1, 2, … , S}

k) E_2={0, 1, 2, … , φ } and

l) $e_{\varphi+1}$ = (1, 1,1, ….,1)′; an (φ +1)-Components column vector of 1's.

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3.4 MODEL ANALYSIS

In our model, we fixed maximum inventory level for fresh items at S and for return items at φ .The inter-arrival time between two sucessive demands are assume to be exponentially distributed with parameter λ for fresh items and δ for return items . Each demand is for exactly one unit for each items. When inventory level reduced to s an order for replanishment is placed. Lead-time is exponentially distributed with parameter γ. When inventory level for the return items reached at φ service will be provided at customer's home.

Now, the infinitesimal generator of the two dimensional Markov process $\{I(t), X(t); t \geq 0\}$ can be defined

$\tilde{A} = \big(a(i,j,k,l)\big); (i,j), (k,l) \in E$

Hence, we get

$$
\tilde{A}(i,j,k,l) = \begin{cases}
\lambda & : \quad i = 1,2,3,\dots S; \quad k = i-1, \quad j = 0,1,2,\dots,\varphi, \quad l = j \\
-(\lambda+\delta+\mu) & : \quad i = s+1, s+2,\dots S; \quad k = i, \quad j = ,1,2,\dots,\varphi-1, \quad l = j \\
-(\lambda+\delta) & : \quad i = s+1, s+2,\dots S; \quad k = i, \quad j = 0, \quad l = j \\
-(\lambda+\mu) & : \quad i = s+1, s+2,\dots S; \quad k = i, \quad j = \varphi, \quad l = j \\
-(\gamma+\lambda+\delta) & : \quad i = 1,2,\dots s; \quad k = i, \quad j = 0, \quad l = j \\
-\mu & : \quad i = 0; \quad k = i, \quad j = 1,2,\dots,\varphi, \quad l = j \\
\delta & : \quad i = 0,1,2,\dots S; \quad k = i, \quad j = 0,1,2,\dots,\varphi-1, \quad l = j+1 \\
\mu & : \quad i = 0,1,2,\dots S; \quad k = i, \quad j = 1,2,\dots,\varphi, \quad l = j-1 \\
\gamma & : \quad i = 0,1,2,\dots s; \quad k = i+Q, \quad j = 0,1,2,\dots,\varphi, \quad l = j
\end{cases}
$$

Now, the infinitesimal generator $\tilde{A}$ can be conveniently express as a partition matrix

$\tilde{A} = (A_{ik})$, where A_{ik} is a $(\varphi+1) \times (\varphi+1)$ sub-matrix which is given by

$$
A_{ik} = \begin{cases}
A_1 & \text{if} \quad k = i-1, i = s+1, s+2,\dots S \\
A_2 & \text{if} \quad k = i, i = s+1, s+2,\dots S \\
A_3 & \text{if} \quad k = i, i = 1,2,\dots s \\
A_4 & \text{if} \quad k = i, i = 0 \\
A_5 & \text{if} \quad k = i-1, i = 1,2,\dots s \\
A_6 & \text{if} \quad k = i+Q, i = 0,1,2,\dots s \\
0 & \text{Otherwise}
\end{cases}
$$

With

$$
A_1 = (a_{ij})_{(\varphi+1)\times(\varphi+1)}
$$
$$
= diag(\lambda\lambda \dots\dots\dots\dots \lambda); \quad where\ (i,j) \to (i-1,j) for\ all\ i = (s+1),(s+2),\dots,S; j = 0,1,2,\dots,\varphi
$$

$$
A_2 = (a_{ij})_{(\varphi+1)\times(\varphi+1)}
$$
$$
= \begin{cases}
(i,j) \to (i,j) & \text{is } -(\lambda+\mu) & \text{for all} & i = (s+1),\dots,S; & j = \varphi \\
(i,j) \to (i,j) & \text{is } -(\lambda+\mu+\delta) & \text{for all} & i = (s+1),\dots,S; & j = 1,2,\dots,\varphi-1 \\
(i,j) \to (i,j) & \text{is } -(\lambda+\delta) & \text{for all} & i = (s+1),\dots,S; & j = 0 \\
(i,j) \to (i,j-1) & \text{is } -\mu & \text{for all} & i = (s+1),\dots,S; & j = 1,2,\dots,\varphi \\
(i,j) \to (i,j+1) & \text{is } \delta & \text{for all} & i = (s+1),\dots,S; & j = 0,1,2,\dots,\varphi-1 \\
Other & elements & are & zero &
\end{cases}
$$

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$A_3 = (a_{ij})_{(\varphi+1)\times(\varphi+1)}$

$$= \begin{cases} (i,j) \to (i,j) & \text{is } -(\lambda+\mu+\gamma) & \text{for all} & i=1,2,\ldots,s; & j=\varphi \\ (i,j) \to (i,j) & \text{is } -(\lambda+\mu+\delta+\gamma) & \text{for all} & i=1,2,\ldots,s; & j=1,2,\ldots,\varphi-1 \\ (i,j) \to (i,j) & \text{is } -(\lambda+\delta+\gamma) & \text{for all} & i=1,2,\ldots,s; & j=0 \\ (i,j) \to (i,j-1) & \text{is } -\mu & \text{for all} & i=1,2,\ldots,s; & j=1,2,\ldots,\varphi \\ (i,j) \to (i,j+1) & \text{is } \delta & \text{for all} & i=1,2,\ldots,s; & j=0,1,2,\ldots,\varphi-1 \\ Other & elements & are & zero & \end{cases}$$

$A_4 = (a_{ij})_{(\varphi+1)\times(\varphi+1)}$

$$= \begin{cases} (i,j) \to (i,j) & \text{is } -(\mu+\gamma) & \text{for all} & i=0; & j=\varphi \\ (i,j) \to (i,j) & \text{is } -(\mu+\delta+\gamma) & \text{for all} & i=0; & j=1,2,\ldots,\varphi-1 \\ (i,j) \to (i,j) & \text{is } -(\delta+\gamma) & \text{for all} & i=0; & j=0 \\ (i,j) \to (i,j-1) & \text{is } -\mu & \text{for all} & i=0; & j=1,2,\ldots,\varphi \\ (i,j) \to (i,j+1) & \text{is } \delta & \text{for all} & i=0; & j=0,1,2,\ldots,\varphi-1 \\ Other & elements & are & zero & \end{cases}$$

$$A_5 = (a_{ij})_{(\varphi+1)\times(\varphi+1)}$$
$$= diag(\lambda\lambda \ldots \ldots \ldots \ldots \lambda);\ where\ (i,j) \to (i-1,j) for\ all\ i = 1,2,\ldots,s; j = 0,1,2,\ldots,\varphi$$

$$A_6 = (a_{ij})_{(\varphi+1)\times(\varphi+1)}$$
$$= diag(\gamma\gamma \ldots \ldots \gamma);\ where\ (i,j) \to (i+Q,j) for\ all\ i = 0,1,2,\ldots,s; j = 0,1,2,\ldots,\varphi$$

So, we can write the partioned matrx as follows:

$$\tilde{A} = \begin{cases} (i,j) \to (i-1,j) is\ A_1 & \forall i = (s+1),(s+2),\ldots,S \\ (i,j) \to (i,j) is\ A_2 & \forall i = (s+1),(s+2),\ldots,S \\ (i,j) \to (i,j) is\ A_3 & \forall\ i = 1,2,\ldots,s \\ (i,j) \to (i,j) is\ A_4 & \forall\ i = 0 \\ (i,j) \to (i-1,j) is\ A_5 & \forall i = 1,2,\ldots,s \\ (i,j) \to (i+Q,j) is\ A_6 & \forall i = 0,1,\ldots,s \end{cases}$$

3.5 Steady State Analysis

It can be seen from the structure of matrix $\tilde{A}$ that the state space E is irreducible.Let the limiting distribution be denoted by $\pi^{(i,j)}$:

$$\pi^{(i,j)} = \underset{t \to \infty}{Lt} \Pr[I(t), N(t) = (i,j)], (i,j) \epsilon E.$$

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Let $\pi = (\pi^{(S)}, \pi^{(S-1)}, \pi^{(S-2)}, \dots\dots, \pi^{(2)}, \pi^{(1)}, \pi^{(0)})$ with

$$\pi^{(k)} = \left(\pi^{(k,\varphi)}, \pi^{(k,\varphi-1)}, \pi^{(k,\varphi-2)}, \dots\dots, \pi^{(k,2)}, \pi^{(k,1)}, \pi^{(k,0)}\right), \forall\, k = 0,1,2, \dots\dots, S.$$

The limiting distribution exists, Satisfies the following equations:

$$\pi\tilde{A} = 0 \;\; and \;\; \sum\sum \pi^{(i,j)} = 1 \dots \dots \qquad (1)$$

The first equation of the above yields the sets of equations:

$$\begin{array}{ll}
\pi^{(1)}A_5 + \pi^{(0)}A_4 = 0 & \\
\pi^{(i+1)}A_5 + \pi^{(i)}A_4 = 0 & : i = 0 \\
\pi^{(i+1)}A_5 + \pi^{(i)}A_3 = 0 & : i = 1,2, \dots\dots, s-1 \\
\pi^{(i+1)}A_1 + \pi^{(i)}A_3 = 0 & : i = s \\
\pi^{(i+1)}A_1 + \pi^{(i)}A_2 = 0 & : i = s+1, s+2, \dots\dots, Q-1 \\
\pi^{(i+1)}A_1 + \pi^{(i)}A_2 + \pi^{(i-Q)}A_6 = 0 & : i = Q, Q+1, \dots\dots, S-1 \\
\pi^{(S)}A_2 + \pi^{(s)}A_6 = 0 &
\end{array}$$

The solution of the above equations(except the last one) can be conveniently express as:

$\pi^{(i)} = \pi^{(0)}\beta_i$; i=0,1,… …,S.

$$\text{Where } \beta_i = \begin{cases}
I & i = 0 \\
-A_5A_4^{-1} & i = 1 \\
(-I)^{i-1}\beta_i(A_5A_4^{-1})^{i-1} & i = 1,2, \dots., s-1 \\
(-I)^{s-1}\beta_i(A_5A_4^{-1})^{s-1}(A_1A_3^{-1}) & i = s \\
(-I)^{i-1}\beta_i(A_5A_4^{-1})^{i-1}(A_1A_3^{-1})(A_2A_1^{-1})^{i-1} & i = s+1, s+2, \dots\dots, Q \\
-\beta_{i-1}(A_2A_1^{-1}) - (A_4A_1^{-1})\beta_{i+Q-1} & i = Q+1, \dots\dots, S
\end{cases}$$

To compute $\pi^{(0)}$, we can use the following equations:

$\pi^{(S)}A_2 + \pi^{(s)}A_6 = 0$ and $\sum \pi^{(k)} e_{K+1} = 1$
Which yeilds respectively
$\pi^{(0)}(\beta_S A_2 + \beta_s A_6) = 0$ and $\pi^{(0)}(I + \sum \beta_i) e_{K+1} = 1$

4 Results

4.1 System Characteristics

(a) Mean inventory level:

(i) The mean inventory level for fresh items: $L_1 = \sum_{i=1}^{S} i \sum_{j=0}^{\varphi} \pi^{(i,j)}$

(ii) The mean inventory level for return items $L_2 = \sum_{j=1}^{\varphi} j \sum_{i=0}^{S} \pi^{(i,j)}$

b) Re-order rate:Re-order rate for fresh items: $R = \lambda \sum_{j=0}^{\varphi} \pi^{(s+1,j)}$

c) Average service rate for return items: $W = \mu \sum_{j=1}^{\varphi} \sum_{i=0}^{S} \pi^{(i,j)}$

d) Average customer lost to the system: $CL = \lambda \sum_{j=0}^{\varphi} \pi^{(0,j)}$

e) Expected total cost: ETC= $c_1 * L_1 + c_2 * L_2 + c_3 * R + c_4 * CL + c_5 * W$;

where, c_1= Holding cost per unit for fresh items,
c_2= Holding cost per unit for return items,
c_3= Replanishement cost per order,
c_4= Service Charge for per unit.

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C_5= Cost of customer lost for per unit.

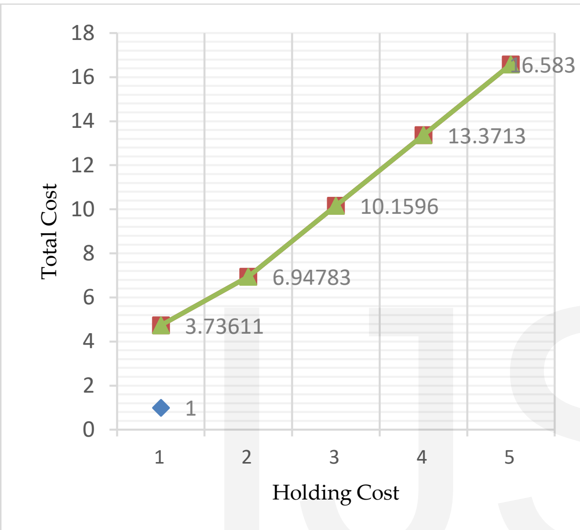
4.2 Numerical Illustrations

Putting, S=5, s=2, φ =3, Q=3, λ =0.35 , δ=0.05, μ =0.01 , γ =0.45 c_1=1.5, c_2=0.70, c_3=0.20, c_4=0.01 ,c_5=0.25 We get

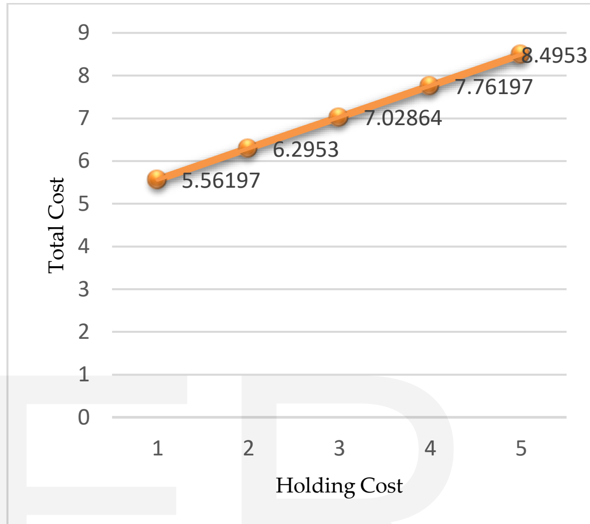
Mean inventory level for fresh items	Mean inventory level for return items	Re-order rate for fresh items	Aaverage service rate	Average customer lost	Expected total cost
3.2117200	**0.7333330**	**0.4862850**	**0.00466667**	**0.0165472**	**5.4323500**

Table 1 Results system characteristics

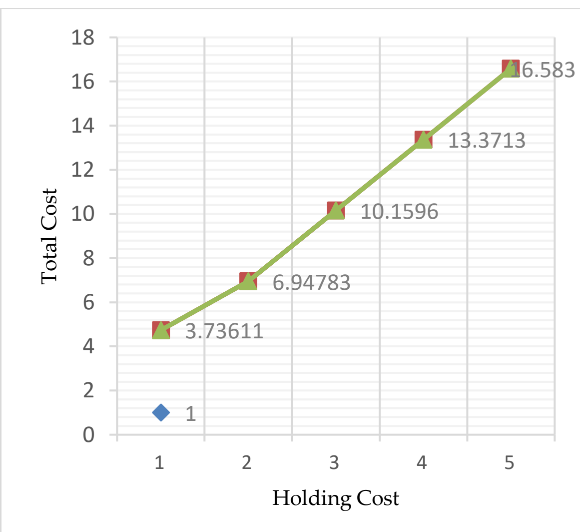
4.3 Graphs of the System



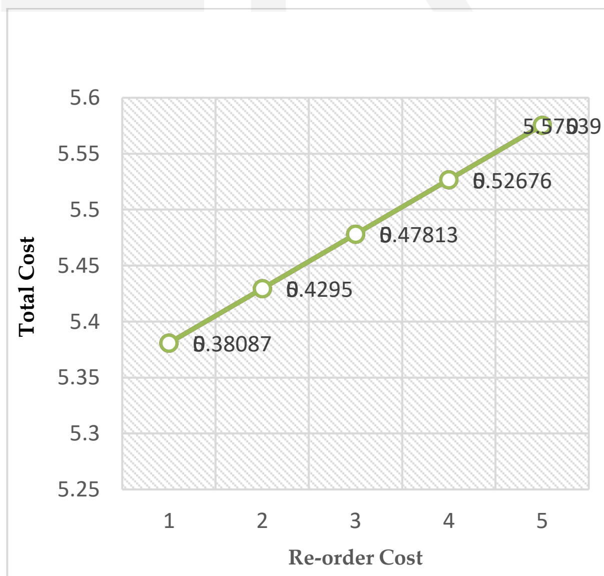
Graph 1 Total Cost vs holding Cost for Fresh Items



Graph 3 Total Cost Vs Holding Cost for Return Items



Graph 2 Total Cost vs holding Cost for Fresh Items



Graph 4 Total Cost vs Re-order Cost

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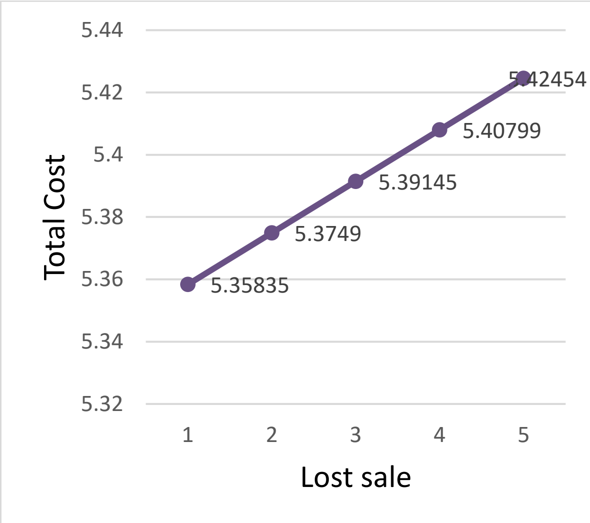
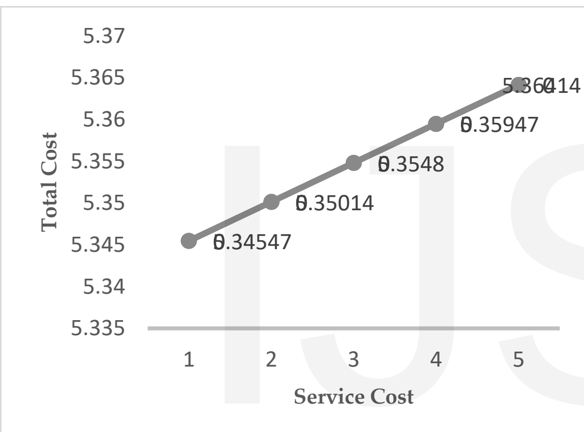


Graph 5 Total Cost Vs Lost Sale



Graph 6 Total Cost vs Service Cost

5 Conclusion

All costs related to inventory system raise total cost. One unit of holding cost for fresh items increase about 3.212 units of total cost where the same cost is about 0.7333 unit for return items. Ordering cost per order increase total cost 0.05 unit. Per unit lost sale is higher than per unit service cost whose increase total cost 0.0165 and 0.0046 units respectively. Since holding cost for fresh items and lost sale are more sensitive, for the betterment of organization we should take care of these costs.

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Appendix

$\pi^{(0,0)}$=0.02521480	$\pi^{(3,0)}$=0.1693730
$\pi^{(0,1)}$=0.01260740	$\pi^{(3,1)}$=0.0846864
$\pi^{(0,2)}$=0.00630370	$\pi^{(3,2)}$=0.0423432
$\pi^{(0,3)}$=0.00315185	$\pi^{(3,3)}$=0.0211716
$\pi^{(1,0)}$=0.03241900	$\pi^{(4,0)}$=0.1369540
$\pi^{(1,1)}$=0.01620950	$\pi^{(4,1)}$=0.0684769
$\pi^{(1,2)}$=0.00810476	$\pi^{(4,2)}$=0.0342385
$\pi^{(1,3)}$=0.00405238	$\pi^{(4,3)}$=0.0171192
$\pi^{(2,0)}$=0.07410060	$\pi^{(5,0)}$=0.0952722
$\pi^{(2,1)}$=0.03705030	$\pi^{(5,1)}$=0.0476361
$\pi^{(2,2)}$=0.01852520	$\pi^{(5,2)}$=0.0238181
$\pi^{(2,3)}$=0.00926258	$\pi^{(5,3)}$=0.0119090

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